The Ion-Acoustic Turbulence in Plasmas with Anisotropically Heated Ions.

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Our paper is devoted to the theoretical discussion of some phenomena connected with the time dependent turbulent plasma heating. The turbulent plasma state with ion-acoustic turbulence was discovered during the sixtieth years of the 20th century because of the pioneering research of the Moscow group in Kurchatov institute and of the Kharkov group in Kharkov Institute of Physics and Technology. This discovery had won recognition in the USSR on the state level. "Discovery no 112: Phenomenon of turbulent heating and anomalous plasma resistance. Application OT-7595 from March 30, 1970, Priority September 9, 1965. Published October 23, 1972. The authors: M.V.Babykin, E.D.Volkov, P.P.Gavrin, B.A.Demidov, E.K.Zavoiskii, L.I.Rudakov, V.A.Skoryupin,V.A.Suprunenko, E.A.Sukhomlin, Ya.B.Fainberg and S.D.Fanchenko" [1]. Moreover this discovery attracted attention of many experimentalists and theoreticians among the plasma physicist all over the world. In particular it is possible to speak about the creation in 20th century of some especial kinetic approach for a description of the ion-acoustic turbulence (IAT) as a new plasma state. This description is based on the Cherenkov interaction of ion-acoustic waves (IAW) with charged plasma particles [2-4] (L.I.Rudakov, L.V.Korablev, 1966; L.M.Kovrizhnykh, 1966, 1967) and on the induced scattering of ion-acoustic waves (IAW) by plasma ions[5,6] (B.Kadomtsev,1964, V.I.Petviashvili, 1963;). Quasilinear approximation permits to predict the anisotropy of the angular distribution of the IAT pulsation that was really observed. The induced scattering of IAW by ions has been permitted to predict the wide frequency spectrum of IAT pulsations: so called Kadomtsev-Petviashvili spectrum. The theoretically predicted quasilinear angular distribution of IAT pulsations obtained by Rudakov and Korablev was nonstationar. The further development of the quasistationary IAT theory was influenced by the hypothesis put forward by Sizonenko and Stepanov [7] about regularization of Rudakov-Korablev spectrum by nonlinear effects. This hypothesis was proved as a theorem later in the theory that took into account simultaneously as the Cherenkov interaction of IAW with charged particles so the induced scattering of IAW by ions [8].

For our talk it is necessary to note that during the long time theory of IAT was based on a model of plasmas with the equal ions charge and mass ratio. The important step on the way of IAT theory of plasmas with unequal ion charge $\left(e_{\alpha}\right)$- to -$\left(m_{\alpha}\right)$mass ratio

$$\frac{e_{\alpha}}{m_{\alpha}} \neq \frac{e_{\beta}}{m_{\beta}}$$

was formulated later [9]. Our paper is devoted to the case of such plasmas.

The importance of the discovery no 112 is connected with the heating of ions that is a very important problem in controlled thermonuclear fusion project. But the theory of the strong turbulent ion heating is consistent only when ion-ion collisions integrals are larger than the collision integrals of ions with the turbulent plasma pulsations and it is possible to speak about Maxwellian ion distribution. Such possibility is absent on the way to the thermonuclear temperature.

So we draw our attention to the case of the negligible two particle ion-ion collisions. But in this case it is difficult to use the existing theory of IAT because the different versions of such theory are based on the hypothesis about the isotropic Maxwellian ion distribution in main region of the velocity space.

On the other side in the case of plasma with a heavy ion admixture the paper [9] demonstrated the anisotropic bi-Maxwellian velocity distribution for heavy ion admixture. Moreover a longitudinal temperature for velocity component antiparallel to the constant strength of electric field $\vec{E}$ which heats plasma and realizes IAT is sufficiently less than transverse temperature for two other velocity components.

Further we use as our supposition [10] for the case of two ion component plasmas when the masses and densities of both species are comparable that the main body of the ion velocity distribution is bi-Maxwellian

$$f_\alpha\left(V_x,V_y,V_z,t\right)=$$
$$=\frac{N_\alpha m_\alpha^{3/2}}{\left(2\pi\right)^{3/2}\kappa_B T_{\perp\alpha}\left(t\right)\left(\kappa_B T_{\|\alpha}\left(t\right)\right)^{1/2}}\exp\left(-\frac{m_\alpha\left(V_x^2+V_y^2\right)}{2\kappa_B T_{\perp\alpha}\left(t\right)}-\frac{m_\alpha V_z^2}{2\kappa_B T_{\|\alpha}\left(t\right)}\right)$$

Here κ_B is the Boltzmann constant, m_α, N_α, $T_{\perp\alpha}\left(t\right)$ and $T_{\|\alpha}\left(t\right)$ are mass, number density, transversal and longitudinal temperature of ions of the α species $\left(\alpha=1,2\right)$. Our supposition about bi-Maxwellian ion distribution is corroborated in our theory. Let us underline that the change in time $T_{\perp\alpha}\left(t\right)$ and $T_{\|\alpha}\left(t\right)$ gives the change of the main body of the ion velocity distribution.

In the limits of such supposition the used later coarsened description of the strong ion heating right up to conditions when the ion-ion collisions are negligible becomes consistent.

This permits us to examine the question about the competition of the electron and ion heating. This new question in the theory of IAT permits us to put another question about the time of the existence of IAT as a phenomenon in the nonisothermal plasma.

It is important to mention that in our approach to IAT we use the so called Coulomb model which takes into account only the Coulomb interaction of charged particles of a plasma. In accordance with this model we can use the following usual expression for the plasma permittivity

$$\varepsilon\left(\omega,\vec{k}\right)=1+\sum_s \frac{4\pi e_s^2}{m_s k^2}\int \frac{1}{\omega-\vec{k}\vec{V}}\vec{k}\frac{\partial f_s}{\partial \vec{V}}d\vec{V} \equiv 1+\delta\varepsilon_e\left(\omega,\vec{k}\right)+\sum_{\alpha=1,2}\delta\varepsilon_\alpha\left(\omega,\vec{k}\right)$$

In the first expression the summation is taken over each kind of the charged plasma particles.

The expounded here variant of the IAT theory uses a number of results of the previous papers which never the less retain their importance.

Similar to our papers [9,11] in which plasmas with two ion species were analyzed, we consider ion-acoustic waves with the spectrum

$$\omega=\omega\left(\vec{k}\right)=V_S k\left(1+k^2 r_{De}^2\right)^{-1/2}$$

Here, $r_{De}=\left(\kappa_B T_e / 4\pi e^2 N_e\right)^{1/2}$ is the electron Debye screening radius; e is the electron charge; N_e is the electron number density; T_e is the electron temperature; $\omega_L=\left(\omega_{L1}^2+\omega_{L2}^2\right)^{1/2}$ $\omega_{L\alpha}=\left(4\pi e_\alpha^2 N_\alpha / m_\alpha\right)^{1/2}$, e_α are the Langmuir frequency, charge of the α-ion species; ω and k are the frequency and the absolute value of the wave vector of ion-acoustic waves, respectively; and $V_S=\omega_L r_{De}$ is the velocity of the ion acoustic wave with long wave length.

Now let us determine our quasistationary approach to the IAT theory. The basic equation of this approach is

$$\gamma\left(\vec{k}\right)\equiv\gamma_e\left(\vec{k}\right)+\sum_{\alpha=1,2}\gamma_\alpha\left(\vec{k}\right)+\gamma_{NL}\left(\vec{k}\right)=0$$

Here $\gamma\left(\vec{k}\right)$ is the increment of ion-acoustic waves which is a sum of the electronic γ_e and ionic γ_α contributions caused by Cherenkov interaction of IAW according to electrons and ions, and the nonlinear ionic contribution γ_{NL} connected with induced scattering of IAW by ions. We must change the description of the last one in comparison with previous. In accordance to formula (13) from [9] for nonlinear contribution γ_{NL} to the damping rate of ion-acoustic waves caused by their induced scattering by ions having nonequal charge-to-mass ratio we have

$$\gamma_{NL}\left(\vec{k}\right)=\int\frac{d\vec{k}}{8\pi}N\left(\vec{k}\right)\frac{\left|\vec{k}-\vec{k}'\right|^2}{\omega_L^4}\left(\frac{\vec{k}\vec{k}'}{kk'}\right)^2\omega\left(\vec{k}\right)\omega\left(\vec{k}'\right)\times$$

$$\times\frac{\partial\delta\left(\omega\left(\vec{k}\right)-\omega\left(\vec{k}'\right)\right)}{\partial\omega(k)}\left(\frac{e_1}{m_1}-\frac{e_2}{m_2}\right)^2\frac{\omega_{L1}^2\delta\varepsilon_2^2\left(0,\vec{k}-\vec{k}'\right)+\omega_{L2}^2\delta\varepsilon_1^2\left(0,\vec{k}-\vec{k}'\right)}{\left[\delta\varepsilon_1\left(0,\vec{k}-\vec{k}'\right)+\delta\varepsilon_2\left(0,\vec{k}-\vec{k}'\right)\right]^2}$$

Here, $N\left(\vec{k}\right)=N(k)\Phi(\cos\theta_k)$ is the axisymmetric distribution of ion-acoustic waves over their wave vectors $\vec{k}$, θ_k is the angle between the wave vector and $e\vec{E}$, $N(k)$ is the distribution of IAT pulsations over their wave numbers value, $\Phi(\cos\theta_k)$ is their angular distribution, and $\delta\varepsilon_\alpha\left(0,\vec{k}-\vec{k}'\right)$ is the contribution of ions of the α species to the static longitudinal plasma permittivity. In accordance to anisotropic bi-Maxwellian velocity distribution of α-species of ions we have

$$\delta\varepsilon_\alpha\left(0,\vec{k}-\vec{k}'\right)=\frac{(4\pi e_\alpha^2 N_\alpha/\kappa_B)}{\left[\left(k_x-k_x'\right)^2T_{\perp\alpha}+\left(k_y-k_y'\right)^2T_{\perp\alpha}+\left(k_z-k_z'\right)^2T_{\|\alpha}\right]}$$

Our change of γ_{NL} description is connected with the approximation of the expression for $\delta\varepsilon_\alpha\left(0,\vec{k}-\vec{k}'\right)$. Our approximation is based on the result of the paper[9] about the heating of small ion admixture when it was demonstrated that the transverse temperature of the ion admixture increases faster than its longitudinal temperature. We use this result as a base of our assumption about analogous result for every kind of ions. This assumption is corroborated in our theory.

So we ignore the longitudinal temperature in comparison with the transversal one in the describing the influence of induced scattering of ion-acoustic waves by ions. Then we use later the following expression

$$\delta\varepsilon_\alpha\left(0,\vec{k}-\vec{k}'\right)=$$

$$=\frac{4\pi e_\alpha^2 N_\alpha}{\kappa_B T_{\perp\alpha}\left[\left(k_x-k_x'\right)^2+\left(k_y-k_y'\right)^2\right]}\equiv\frac{1}{r_{\perp\alpha}^2\left[\left(k_x-k_x'\right)^2+\left(k_y-k_y'\right)^2\right]}$$

instead of the expression $\delta\varepsilon_\alpha\left(0,\vec{k}-\vec{k}'\right)=r_{D\alpha}^{-2}\left(\vec{k}-\vec{k}'\right)^{-2}$ in the theory with Maxwellian ion distributions. Here $r_{D\alpha}=\left(\kappa_B T_\alpha / 4\pi e_\alpha^2 N_\alpha\right)$ is the Debye radius of ions with temperature T_α.Therefore our proposed approximate theory of the nonlinear influence of scattering by ions differs from the IAT theory proposed about twenty years ago[9] in that, here, the ion Debye radii are replaced with the transverse ion radii

$$r_{D\alpha}=\sqrt{\frac{4\pi e_\alpha^2 N_\alpha}{\kappa_B T_\alpha}}\rightarrow r_{\perp\alpha}=\sqrt{\frac{4\pi e_\alpha^2 N_\alpha}{\kappa_B T_{\perp\alpha}}}$$

Then after we so coarsen the description of the induced scattering of IAW by ions we can write

$$\gamma_{NL}\left(\vec{k}\right)=\int\frac{d\vec{k}}{8\pi}N\left(\vec{k}\right)\frac{\left|\vec{k}-\vec{k}'\right|^2}{\omega_L^4}\left(\frac{\vec{k}\vec{k}'}{kk'}\right)^2\frac{\omega_{L1}^2 r_{\perp1}^4+\omega_{L2}^2 r_{\perp2}^4}{\left[r_{\perp1}^2+r_{\perp2}^2\right]^2}\omega\left(\vec{k}\right)\times$$

$$\times\frac{\partial\delta\left(\omega\left(\vec{k}\right)-\omega\left(\vec{k}'\right)\right)}{\partial\omega\left(k\right)}\left(\frac{e_1}{m_1}-\frac{e_2}{m_2}\right)^2\omega\left(\vec{k}'\right)$$

The simple replacement of $r_{D\alpha}$ by $r_{\perp\alpha}$ permits to use the result of the previous theory for the distribution of ion-acoustic pulsations on the absolute values of the wave vectors immediately $N\left(k\right)=\sqrt{\frac{\pi}{2}}\frac{\omega_L^6 r_{De}^5}{\omega_{Le}}\frac{1}{\left(\omega_{L1}^2 r_{\perp1}^4+\omega_{L2}^2 r_{\perp2}^4\right)}\frac{\left(r_{\perp1}^2+r_{\perp2}^2\right)Y\left(kr_{De}\right)}{\left[\left(e_1/m_1\right)-\left(e_2/m_2\right)\right]^2}$

$$Y\left(x\right)=\frac{x^{-4}}{\left(1+x^2\right)}[\ln\left(\frac{\sqrt{1+x^2}+1}{x}\right)-\frac{1}{\left(1+x^2\right)^{1/2}}-\frac{1}{3\left(1+x^2\right)^{3/2}}]$$

Also such simple replacement gives us the so called turbulent Knudsen number

$$K_{N2}=6\pi^2\frac{\left|e\right|N_e\omega_{Le}^2}{\omega_L^8 r_{De}}\left(\frac{e_1}{m_1}-\frac{e_2}{m_2}\right)^2\frac{\left(\omega_{L1}^2 r_{\perp1}^4+\omega_{L2}^2 r_{\perp2}^4\right)}{\left(r_{\perp1}^2+r_{\perp2}^2\right)^2}E\equiv\frac{E}{E_{N2}}$$

which in comparison with one permits to distinguish the limits of the weak and strong electric fields.

The formula for the electronic increment of IAW in the basic equation is usual one [2-4,8,9]. Here we give as an illustration such formula for the case of the Maxwellian electron distribution [9]

$$\gamma(k,\theta_k)=\gamma_S(k)\left(\frac{\omega}{kV}\right)^3(-\left(\frac{\omega}{kV_S}\right)+\frac{2}{\pi}cos\theta_k\times$$

$$\int_0^{\sin\theta_k}\frac{d\xi}{\left(\sin^2\theta_k-\xi^2\right)^{1/2}}\left[\frac{\nu_E}{\nu_2\left(\sqrt{1-\xi^2}\right)}+\frac{\nu_1\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}\,\nu_2\left(\sqrt{1-\xi^2}\right)}\right])$$

where

$$\nu_n(y)=\int_0^\infty\frac{k^3dk}{4\pi^2}\left(\frac{\omega}{kV_S}\right)^{4-n}\int_{-1}^{+1}\frac{dx}{\left(y^2-x^2\right)^{1/2}}\left(\frac{x}{y}\right)^n\frac{\omega N(k,x)}{N_e\sqrt{m_e\kappa_BT_e}},$$

$$\nu_E=(9\pi/8)^{1/2}|e|E/m_eV_S,\qquad \gamma_S(k)=(\pi/8)^{1/2}kV_S(\omega_L/\omega_{Le}).$$

The last term in our discussion of the basic Eqn. $\gamma(\vec{k})=0$ is the ion decrement of IAW, connected with the Cherenkov interaction of waves with ions. The ion Cherenkov interaction takes into account only Landau damping of IAW. So for the ratio between the ion and electron damping rates of IAW we can use

$$\delta\left(\cos^2\theta_k\right)=\sum_{\alpha=1,2}\delta_\alpha\left(\cos^2\theta_k\right)=$$

$$=\sum_{\alpha=1,2}\frac{\omega_{Le}}{\omega_{L\alpha}}\left(\frac{r_{De}^2}{r_{\perp\alpha}\sin^2\theta_k+r_{\|\alpha}\cos^2\theta_k}\right)^{1/2}\exp\left(-\frac{\omega_L^2r_{De}^2}{2\omega_{L\alpha}^2\left(r_{\perp\alpha}\sin^2\theta_k+r_{\|\alpha}\cos^2\theta_k\right)}\right)$$

Here ω_{Le} is the electron Langmuir frequency.

Now we can say that the basic equation permits us to obtain not only $N(k)$ but permits us to obtain the following nonlinear Abel-type integral equation for the angular distribution of IAT pulsations $\Phi(\cos\theta_k)$ (compare [9]):

$$\int_0^x dt\left[\frac{t}{x^2}\left(1+\phi\left(x^2\right)+\Delta\left(x^2\right)\right)-1\right]\times\frac{t\Phi(t)}{\left(x^2-t^2\right)^{1/2}}=\frac{K_{N2}}{\lambda}x^2,\qquad 0\le x\le 1$$

where

$$\phi\left(x^2\right)=\frac{1}{2}(M_0-M_2+\left(6M_1-10M_3\right)x^4+$$
$$+\left(6M_1-10M_3\right)x^4-\left(M_o+6M_1-3M_2-8M_3\right)\mathrm{x}^2+$$
$$+\left(M_0-3M_2\right)x^2\left(1-x^2\right)^{1/2}\ln\left(\frac{1+\sqrt{1+x^2}}{x}\right))$$

is the function describing the nonlinear influence of turbulence on its angular distribution because of $M_n=\int_0^1 x^n\Phi\left(x\right)dx$ and $\lambda\simeq 0.5$. The new angular dependence is connected with the function

$$\Delta\left(\cos^2\theta_k\right)=\sum_\alpha\Delta_\alpha\left(\cos^2\theta_k\right)=$$
$$=\sum_\alpha\cos\theta_k\frac{d}{d\theta_k}\int_0^{\theta_k}\frac{\sin\theta\delta_\alpha\left(\cos^2\theta\right)d\theta}{\left(\cos^2\theta-\cos^2\theta_k\right)}$$

resulting from anisotropy of the ion velocity distribution.

In the limit $K_{N2}/\lambda\gg 1$ the solution of nonlinear Abel-type integral equation was obtained previously [9]

$$\Phi\left(x\right)=\frac{2K_{N2}}{\pi\lambda x^2}\frac{d}{dx}\int_0^x\frac{\varsigma^5 d\varsigma}{\sqrt{x^2-\varsigma^2}\phi\left(\varsigma^2\right)}$$

This is a case of the strong electric field where the function $\phi\left(\varsigma^2\right)$ contains $M_n=\sqrt{2K_{N2}/\pi\lambda}b_n$ where $b_0=2.47;\ b_1=1.84;\ b_2=1.44;\ b_3=1.17.$

Further in the discussion of the results which are connected with the turbulent plasma heating we limit ourselves to this discussed case of the strong electric field.

The results of the theory of IAT angular distribution permit us to look at the kinetic equations which describe the distribution of the main group of ions in the ion velocity space. These equations were derived in papers [9,12]. But because of these kinetic equations describe the evolution to bi-Maxwellian distribution we propose our replacement $r_{D\alpha}\to r_{\perp\alpha}$. In the $\left(K_{N2}/\lambda\right)\gg 1$ limit in accordance with [12] we have

$$\frac{\partial f_\alpha}{\partial t} = d_\alpha \left[A_\perp \left(\frac{\partial^2 f_\alpha}{\partial V_x^2} + \frac{\partial^2 f_\alpha}{\partial V_y^2} \right) + A_\| \frac{\partial^2 f_\alpha}{\partial V_z^2} \right]$$

where

$$d_\alpha = \frac{|e| E N_e \omega_L r_{De} \omega_{L\alpha}^2 r_{\perp\alpha}^4}{m_\alpha N_\alpha \left(\omega_{L1}^2 r_{\perp 1}^4 + \omega_{L2}^2 r_{\perp 2}^4 \right)}$$

and $A_\perp \cong 0.6$, $A_\| \cong 0.15$. The solution of these ion kinetic equations gives us the anisotropic bi-Maxwellian velocity distributions of both ion components. The transversal and longitudinal temperatures evolve according to the equations ($\alpha = 1, 2$)

$$\frac{d(\kappa_B T_{\perp(\|)\alpha})}{dt} = 2 m_\alpha d_\alpha A_{\perp(\|)}$$

Therefore our disregard of $r_{\|\alpha}$ in comparison with $r_{\perp\alpha}$ in the case of the strong electric fields shows us the accuracy to within 25% of the description of the induced IAW scattering by ions. Such accuracy is better in the case of the weak electric fields [10]. We can write the following equation for the transversal temperatures

$$\frac{dT_{\perp 1}}{dT_{\perp 2}} = \frac{m_1 d_1}{m_2 d_2}$$

The last equation in the limit of the strong electric field looks simply

$$\frac{dT_{\perp 1}}{dT_{\perp 2}} = \frac{m_2 e_2^2 N_2^2}{m_2 e_2^2 N_2^2} \times \frac{T_{\perp 1}^2}{T_{\perp 2}^2}$$

Then we have the simple solution

$$T_{\perp 1}^{-1}(t) - T_{\perp 1}^{-1}(t_0) =$$

$$= \frac{m_2 e_2^2 N_2^2}{m_1 e_1^2 N_1^2} \left(\frac{1}{T_{\perp 2}(t)} - \frac{1}{T_{\perp 2}(t_0)} \right)$$

If we have at the initial time

$$\frac{T_{\perp 2}(t_0)}{T_{\perp 1}(t_0)} = \frac{m_2 e_2^2 N_2^2}{m_1 e_1^2 N_1^2}$$

then this ratio remains valid later at $t \geq t_0$.

Further we should like to discuss as an illustration the simple example of the equal ions temperatures heating when $T_{\perp 1}(t_0) = T_{\perp 2}(t_0) = T_{\perp}(t_0)$ and $m_1 e_1^2 N_1^2 = m_2 e_2^2 N_2^2$. In opposite case $m_1 e_1^2 N_1^2 \neq m_2 e_2^2 N_2^2$ if we assume the monotonic growing up $T_{\perp 1}(t)$ then we have $T_{\perp 1}(t \to \infty) \to \infty$ and $\frac{T_{\perp}(t_0)}{T_{\perp 2}(t \to \infty)} = \frac{m_2 e_2^2 N_2^2}{m_2 e_2^2 N_2^2 - m_1 e_1^2 N_1^2}$. So if $m_2 e_2^2 N_2^2 > m_1 e_1^2 N_1^2$ then $T_{\perp 2}(t)$ strives to its limiting saturation value.

To discuss the heating of two ion components with the equal temperatures in the regime without the saturation we can use the kinetic equations

$$\frac{\partial f_\alpha}{\partial t} = \bar{d}_\alpha \left[A_\perp \left(\frac{\partial^2 f_\alpha}{\partial V_x^2} + \frac{\partial^2 f_\alpha}{\partial V_x^2} \right) + A_\parallel \frac{\partial^2 f_\alpha}{\partial V_z^2} \right]$$

where

$$\bar{d}_\alpha = \frac{|e| E N_e \omega_L e_\alpha^2 N_\alpha^2}{\sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3} r_{De}(t)$$

The solutions of these equations for the space uniform plasmas are bi-Maxwellian distributions with the transversal and longitudinal temperatures which are subjected to equations

$$\frac{d\left(\kappa_B T_{\perp(\parallel)\alpha}\right)}{dt} = 2 m_\alpha A_{\perp(\parallel)} \bar{d}_\alpha(t)$$

The right side part of these two equations is time dependent because of the time dependence of the electron temperature $T_e(t)$. In the case of the strong electric field the growing up of $T_e(t)$ is practically connected only with the turbulent Joule heating

$$\frac{3}{2} N_e \frac{d(\kappa_B T_e)}{dt} \cong \sigma E^2$$

According to the paper [11] in the limit of the strong electric field the turbulent electrical conductivity can be written as

$$\sigma \cong 1.2 \frac{|e| N_e V_S}{E} \sqrt{K_{N2} / \lambda}$$

Then we have

$$\frac{d\left(\kappa_B T_e\right)^{3/4}}{dt} = 0.6 \frac{|e| E \omega_L \sqrt{R}}{\left(4\pi e^2 N_e\right)^{1/4}} \quad .$$

For the ions heating with equal temperatures and without saturation we have

$$R = \frac{K_{N2}}{\lambda} r_{De} \to \tilde{R} = \frac{12\pi^2 |e| N_e E \omega_{Le}^2 \bar{\omega}^2}{\omega_L^8} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right)^2,$$

where

$$\bar{\omega}^2 = 4\pi \sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3 \left(\sum_{\alpha=1,2} m_\alpha N_\alpha \right)^{-2} \quad .$$

Therefore we can write the following time dependence of the electron temperature

$$\kappa_B T_e(t) = (\left[\kappa_B T_e(t_0)\right]^{3/4} + \left(\frac{E^2}{4\pi N_e} \right)^{3/4} g_e^{3/4} \left[\omega_L (t - t_0)\right])^{4/3}$$

where $$g_e = \left[3.4 \frac{\omega_{Le}^2 \bar{\omega}^2 \omega_H^4}{\omega_L^8} \left(\frac{Z_1}{A_1} - \frac{Z_2}{A_2} \right)^2 \right]^{2/3},$$

and $Z_\alpha = e_\alpha / |e|$, $A_\alpha = m_\alpha / m_H$, m_H is the hydrogen atom mass $\omega_H^2 = 4\pi e^2 N_e / m_H$.

The regime of the strong electron heating $T_e(t) >> T_e(t_0)$ is described by formula

$$N_e \kappa_B T_e(t) = \frac{E^2}{4\pi} g_e \left[\omega_L (t - t_0)\right]^{4/3}$$

Now we can obtain the explicit time dependence of the ion temperatures

$$N_\alpha \kappa_B T_{\perp(\parallel)\alpha}(t) = N_\alpha \kappa_B T_{\perp(\parallel)\alpha}(t_0) + \frac{1.2 A_{\perp(\parallel)} m_\alpha e_\alpha^2 N_\alpha^3}{\sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3} \sqrt{g_e} \frac{E^2}{4\pi} \left[\omega_L (t - t_0)\right]^{5/3}.$$

On the other side the electron temperature grows as $\left[\omega(t - t_0)\right]^{4/3}$. We see that the speed of the growing up of the ions temperature can exceed the speed of the electron temperature growing up for enough time. Moreover this tendency permits to see the possibility to violate the condition of the ion temperature smallness in comparison with the electron temperature. The last violation is the violation of the condition of the ion acoustic wave existence and therefore the existence of the ion acoustic turbulence.

$$\frac{T_e(t)}{T_{\perp\alpha}(t)} = (\left[\kappa_B T_e(t_0)\right]^{3/4} + \left(\frac{E^2}{4\pi}\right)^{3/4} g_e^{3/4} \times \left[\omega_L(t-t_0)\right])^{4/3} \times$$

$$(\kappa_B T_{\perp\alpha}(t_0) + \frac{E^2}{4\pi}\sqrt{g_e}1.2A_\perp \frac{m_\alpha e_\alpha^2 N_\alpha^2 N_e}{\sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3}\left[\omega_L(t-t_0)\right]^{5/3})^{-1}$$

This ratio of temperatures of electrons and ions permits to obtain numerical estimation of the existence time of the strong field regime of the turbulent plasma heating. More transparent form we have in the case of the strong electron and ion heating ($T_e(t) >> T_e(t_0)$, $T_{\perp\alpha}(t) >> T_{\perp\alpha}(t_0)$)):

$$\left[T_e(t)/T_{\perp\alpha}(t)\right] = (N_\alpha\sqrt{g_e}\sum_\beta m_\beta e^2 N_\beta^3)(1.2A_\perp N_e m_\beta e_\alpha^2 N_\alpha^2\left[\omega_L(t-t_0)\right]^{1/3})^{-1}$$

We can try to obtain the necessary estimation after we set this ratio to five. Then we have

$$\left[\omega_L(t_f - t_0)\right] = g_e^{3/2}\left[\frac{\sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3}{3.6 m_\alpha e_\alpha^2 N_\alpha^2 N_e}\right]^3$$

Let us look at deuterium-tritium plasma as an example in the case of the strong electric field and in the regime of the two component heating without saturation. Then let obtain the estimation of the existence time of discussed strong field regime of the turbulent deuterium-tritium plasma heating in the case of plasma with the equal ions temperatures and with $(N_d/N_t) = \sqrt{3/2}$. Then $g_e \cong 66$ and $\omega_L(t_f - t_0) \cong 11.5$. During this time the electrons and ions temperatures increase in equal times

$$\frac{T_e(t_f)}{T_e(t_0)} = \frac{T_{\perp\alpha}(t_f)}{T_{\perp\alpha}(t_0)} = 136\frac{E^2}{N_e\kappa_B T_e(t_0)}$$

As for such deuterium-tritium plasma we have

$$K_{N2}(t_0) \cong 210\frac{E}{\sqrt{N_e\kappa_B T(t_0)}}$$

then we can rewrite the previous relation as

$$\frac{T_e(t_f)}{T_e(t_0)}=\frac{T_{\perp\alpha}(t_f)}{T_{\perp\alpha}(t_0)}=\left(\frac{K_{N2}(t_o)}{18}\right)^2$$

In particular this relation permits to see that during the time of the turbulent heating the temperatures of electrons and ions can grow up to about a hundred times when $K_{N2}\approx 180$.

The presented here phenomenon of the finite time existence of IAT is important for understanding of IAT. But it is necessary to underline that this phenomenon is discussed in the frame of the model of IAT that is related to plasmas with the different ions with unequal charge-to-mass ratio. Therefore there is a problem to understand the possibility of the analogous phenomenon in the bounds of a simpler model of plasmas with one kind of ions. Moreover this phenomenon bears a relation to theory of papers [9] and [10] only in the case of the neglect of the Cherenkov interaction of ions with the turbulent pulsations. In addition the numerical estimation of the existence time of IAT can be changed owing to the use of the more precise angular distribution in comparison with asymptotic approximation of the strong field that we use in our discussion. So the aim of my discussion of the problem of the turbulent plasma heating is to attract more attention to the question of the time duration of the IAT existence.

The other result of the plasma heating manifests in the properties of the turbulent conductivity σ, which is usually presented in the theory of IAT as a nonlinear function of the electric field strength. Let us describe here the appropriate result for the case of the strong field when according to (44) and (12) there is the explicit dependence $\sigma\approx E^{-1/2}$ which is usually discussed in the theory of IAT. However besides this explicit dependence the turbulent conductivity depends on electron temperature which because of plasma heating is determined by the electric field. The obvious simple example is the plasma heating in the strong field when ions are heated in the regime without the saturation. The additional simplification arises in the conditions of the strong plasma particle heating $T_e(t) >> T_e(t_0)$, $T_{\perp\alpha}(t) >> T_{\perp\alpha}(t_0)$. Then we can write down

$$\sigma = 0.30 g_e^{3/4} (N_e \kappa_B T_e(t) / E^2)^{1/4} \omega_L$$

Using the previously discussed electron temperature time dependence we have

$$\sigma = 0.16 g_e \left[\omega_L (t - t_0) \right]^{1/3} \omega_L .$$

In particular case of the deuterium-tritium plasma discussed previously we have $\sigma \cong 11 \left[\omega_L (t - t_0) \right]^{1/3} \omega_L < 24 \omega_L$. Let us underline that in accordance this discussion the turbulent conductivity is independent of the electric field strength. In this connection we can speak about the reduction of the nonlinear dependence of turbulent conductivity to the field independent expression of the linear theory of electricity. But this linear theory describes the time dependent electric field because of the time dependence of the turbulent conductivity.

Our model uses the time independent electric field strength. In the case of the quasistationary time change of the electric field we can see that the nonlinear turbulent conductivity can be connected with the following form

$$\{ \int_{t_0}^{t} d(\omega_L t') E^{3/2}(t') \}^{1/3} E^{-1/2}$$

The last nonlinear field dependence in general is weaker than $\sigma \approx E^{-1/2}$ that we have without the influence of the plasma heating. This discussed effect is analogous one to the effect discussed previously [13] in connection with the possible understanding of Demidov-Elagin-Fanchenko effect [14].

Conclusion.

1. The coarsened description of the stimulated scattering of the ion acoustic waves on ions is the base of the delivered rough variant of the IAT theory. This permits to work out the description of the plasma particles heating conditions when the particles collisions are not important.

2. The possibility to describe the strong plasma particles heating permits to put the question about the time duration of the existence of IAT plasma state and to see how the nonlinear electric field dependent turbulent conductivity turns into the field independent conductivity.

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